Code No. 2860 / CDE

FACULTY OF SCIENCE

M.Sc. Final (CDE) Examination, July 2018
Subject: Mathematics

Paper – I
Complex Analysis

Time: 3 Hours **Max.Marks: 80**

PART – A (5x4 = 20 Marks)
[Short Answer Type]

Note: Answer any five of the following questions. Each question carries 4 marks.

1 Find the fixed points of the Mobins transformation w = $\frac{2z}{3z-1}$.

2 Prove that the reflection z $\rightarrow \bar{z}$ is not a linear transformation.

3 Compute $\int_{|z|=r} x\, dz$, for the positive sense of the circle.

4 Suppose f(z) and g(z) are analytic in a region Ω, and if f(z) = g(z) on a set which has an accumulation point in Ω, then prove that f(z) = g(z) $\forall$ z $\varepsilon\, \Omega$.

5 Find the poles and residues of the function f(z) = cot z.

6 How many roots of the equation $z^4 - 6z + 3 = 0$ have their modulus between 1 and 2.

7 Prove that for |z| < 1, $(1+z)(1+z^2)(1+z^4)(1+z^8) \ldots = \frac{1}{1-z}$.

8 What are the residues of Γ(z) at the poles z = -n?

PART – B (5x12 = 60 Marks)
[Essay Answer Type]

Note: Answer all the following questions by using internal choice.
Each question carries 12 marks.

9 a) State and prove Abel's theorem.

OR

b) Show that the smallest positive period of e^{iz} is 2®.

10 a) Compute $\int_{|z|=1} |z-1|.|dz|$.

OR

b) Suppose f(z) is analytic on a closed curve¥ . Show that $\int_{¥} \overline{f(z)}\, f'(z)\, dz = 0$.

11 a) State and prove Cauchy residue theorem.

OR

b) Evaluate $\int_{o}^{\infty} \frac{x \sin x}{(x^2 + a^2)} dx$, where a is real.

12 a) Prove that $(2®)^{\frac{n-1}{2}} \Gamma(z) = n^{z-\frac{1}{2}} \Gamma\left(\frac{3+1}{n}\right) \ldots \Gamma\left(\frac{z+n-1}{n}\right)$.

OR

b) State and prove Legendre's duplication formula.

13 a) State and prove Poisson's formula.

OR

b) State and prove Jensen's formula.

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Code No. 2865 / CDE / O / BL

FACULTY OF SCIENCE

M.Sc. Final (CDE) Examination, July 2018
Subject: Mathematics
Paper – I
Topology & Functional Analysis

Time: 3 Hours **Max.Marks: 100**

PART – A
[Short Answer Type]

Note: Answer any five of the following questions, choosing atleast two from each part. All questions carry equal marks.

1 a) State and prove Lindelof's theorem.

b) Let X be any non-empty set and let S be an arbitrary class of subsets of X. Then prove that S can serve as an open sub-base for a topology on X, in the sense that the class of all unions of finite intersections of sets in S is a topology on X.

2 a) Show that a metric space is compact if and only if it is complete and totally bounded.

b) Show that a closed subspace of a complete metric space is compact if and only if it is totally bounded.

3 a) Prove that in a sequentially compact metric space, every open cover has a Lebesgue number.

b) Prove that every sequentially compact metric space is totally bounded.

4 State and prove Urysohn's imbedding theorem.

5 If X is an arbitrary topological space then prove the following:

i) Each point in X is contained in exactly one component of X.

ii) Each connected subspace of X is contained in a component of X.

iii) A connected subspace of X which is both open and closed in a component of X

iv) Each component of X is closed.

PART – B
[Essay Answer Type]

6 If N and N' are normed linear spaces then prove that B(N, N') = set of all continuous linear transformations of N into N' is itself a normed linear space with respect to point wise linear operations and the norm defined by $||T|| = \text{Sup}\{||T(x)|| : ||x|| \leq 1\}$. Further, if N' is a Banach space then prove that B(N, N') is also a Banach space.

7 Let M be a linear subspace of a normed linear space N and let f be a functional defined on M. If x_o is a vector not in M and if $M_o=M+[x_o]$ is a linear subspace spanned by M and x_o then prove that f can be extended to a functional f_o defined on M_o such that $||f_o|| = ||f||$.

8 a) If M and N are closed linear subspaces of a Hilbert space H such that $M \perp N$ then prove that M+N is also closed.

b) If M is a closed linear subspace of a Hilbert space H, then prove that $H=M \oplus M^{\perp}$.

9 a) If $\{e_i\}$ is an orthonormal set in a Hilbert space H, then prove that $\Sigma|(x, e_i)|^2 \leq ||x||^2$ for every vector x in H.

b) Discuss Gram-Schmidt orthogonalization process.

10 a) If P is a projection on H with range M and Null space N, then prove that $M \perp N$ if and only if P is self-adjoint and in this case $N=M^{\perp}$.

b) If $P_1, P_2, \ldots, P_n$ are the projections on closed linear subspaces $M_1, M_2, \ldots, M_n$ of H then prove that $P = P_1+P_2+ \ldots +P_n$ is a projection if and only if the P_i's are pair-wise orthogonal and in this case prove that P is projection on $M = M_1 + M_2 + \ldots + M_n$.

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Code No. 2875/CDE

FACULTY OF SCIENCE
M. Sc. (Final) (CDE) Examination, July 2018

Subject : Statistics

Paper – I : Statistical Inference

Time : 3 Hours **Max. Marks: 80**

Note : Answer any five questions from Part–A, each question carries 4 marks. Answer all questions in Part–B, each question carries 12 marks.

PART – A (5 x 4 = 20 Marks)
(Short Answer Type)

1 Distinguish between (a) Type I and Type II errors and (b) randomized and non-randomized tests.
2 Define unbiased test and UMPU test of size α. Prove that UMP test is UMPU.
3 Describe Wald's SPRT. Give expressions for OC and ASN functions.
4 Explain the need for SPRT with an example.
5 Define one-sample U-statistic. Obtain its asymptotic variance.
6 What are contingency tables? Explain Chi-square test for independence.
7 Define Bayes rule and Bayes risk.
8 Explain squared error and absolute error loss functions.

PART – B (5 x 12 = 60 Marks)
(Essay Answer Type)

9 (a) Given a random sample of size n from a distribution with pdf $f_\theta(x)$ having MLR in $T(x)$, obtain an UMP size α test for testing $H_0 : \theta \le \theta_o$ vs $H_1 : \theta > \theta_o$.

OR

(b) If $X_1,\ldots,X_n$ is a random sample from $N(\mu, \sigma^2)$, μ and σ^2 unknown, obtain likelihood ratio test for testing $H_o : \sigma^2 = \pm_o^2$ against $H_1 : \sigma^2 \neq \pm_o^2$

10 (a) A random variable X has pmf $p(x : ¯) = \begin{cases} ¯/4 & if \quad x = -1 \\ 1 - ¯ & if \quad x = 0 \\ 3¯/4 & if \quad x = 1 \end{cases}$

find h(θ) such that $E_{¯} = \left\{ \dfrac{p(X : 3/4)}{p(X : 1/4)} \right\} = 1$

and hence obtain the approximate expression for OC function of the SPRT for testing $H_o : \theta = 1/4$ vs $H_1 : \theta = 3/4$.

OR

(b) Prove that SPRT terminates with probability one.

11 (a) Bring out the difference between sign test and Wilcoxon signed rank test. Explaining one-sample location problem, give the test procedure of Wilcoxon signed rank test with all the details of the test.

OR

(b) Define K-sample problem. Explain Ansari –Bradley and Kruskal-Wallis test for one way layout problems. Compare the two tests.

12 (a) Define Pitman ARE. Derive ARE between median test and Wilcoxon-rank sum test. Give your comments.

OR

(b) Define conjugate family of distributions. Prove that the normal distribution is a conjugate family.

13 (a) Explain SPRT procedure for Poisson distribution and also obtain its OC and ASN functions.

OR

(b) State and prove Neyman person Lemma.

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Code No. 2861 / CDE

FACULTY OF SCIENCE

M.Sc. Final (CDE) Examination, July 2018

Subject: Mathematics

Paper – II
Measure Theory

Time: 3 Hours **Max.Marks: 80**

PART – A (5x4 = 20 Marks)
[Short Answer Type]
Note: Answer any five of the following questions. Each question carries four marks.

1 Define an algebra and a $\pm -$algebra of subsets of X. Give an example of each. Also give an example of an algebra of sets which is not a $\pm -$algebra.

2 State and prove bounded convergence theorem.

3 Define positive variation sum, negative variation sum and total variation sum of a real valued function f defined on [a,b] with respect to a partition P of [a,b]. Also prove that n+p = t.

4 State and prove Minkowski's inequality for $1 < p < \infty$.

5 Suppose $f_1, f_2, \dots f_n$ are measurable functions. Prove that max $(f_1, f_2, \dots f_n)$ and min $(f_1, f_2, \dots f_n)$ are also measurable functions.

6 Suppose (x, , «) is a measure space and E ϵ . Suppose f and g are non-negative measurable functions defined on E prove that $\int_E (af+bg)du = a \int_E f\, du + b \int_E g\, du$ ∀ non-negative constants a,b.

7 Suppose E_1 and E_2 are μ^*-measurable. Prove that $E_1 \cup E_2$ is also μ^*-measurable.

8 Define inner measure μ_* induced by a measure μ on an algebra of subsets of X. Prove that $\mu_*(E) \leq \mu^*(E)\ \forall\ E \subseteq x$.

PART – B (5x12 = 60 Marks)
[Essay Answer Type]
Note: Answer all the following questions by using internal choice.
Each question carries 12 marks.

9 a) Prove that there exists a non-measurable subset of IR.

OR

b) State and prove Fotou's lemma.

10 a) Suppose f is a real valued function defined on [a,b]. Prove that f is a function of bounded variation on [a,b] if and only if f can be expressed as a difference of two monotonically increasing functions on [a,b].

OR

b) Suppose f is integrable on [a,b] and F(x) = $\int_a^x f(t)dt \ \forall x \in [a,b]$. Prove that f is a continuous function of bounded variation on [a,b].

11 a) Suppose (x,) is a measurable space and f is an extended real valued function defined on E ϵ . Prove that the following are equivalent

i) {x ϵ E : f(x) > ~ } ϵ $\forall$ ~ ϵ IR

ii) {x ϵ E : f(x) ≥ ~ } ϵ $\forall$ ~ ϵ IR

iii) {x ϵ E : f(x) <~ } ϵ $\forall$ ~ ϵ IR

iv) {x ϵ E : f(x) ≤ ~ } ϵ $\forall$ ~ ϵ IR

OR

b) State and prove Lebesque decomposition theorem.

12 a) Prove that class of all μ^*-measurable sets forms a± – algebra.

OR

b) Suppose μ^* is a Caratheodors outer measure on p(x) with respect to Γ prove that every function in Γ is μ^*-measurable.

13 a) Prove that intervals of the form (a,∞) are measurable. Hence prove that every Borel set is measurable.

OR

b) Suppose E is a set of positive measure with respect to a signed measure¬$_i$ prove that E contains a positive set A with ¬ (A) > 0.

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Code No. 2866 / CDE / O / BL

FACULTY OF SCIENCE

M.Sc. Final (CDE) Examination, July 2018

Subject: Mathematics

Paper – II
Measure & Integration

Time: 3 Hours **Max.Marks: 100**

Note: Answer any five questions. Each question carries 20 marks.

1 a) Given any collection ϱ e of subsets of a given set X. Prove that there exists the smallest ± −algebra of subsets of X containing ϱ.

b) Prove that the smallest ± −algebra of subsets of R containing the class of all intervals in R is the class of all Borel sets.

2 Prove that the class $\mathcal{M}$ of all Lebesque measurable sets is a ± −algebra of sets in R.

3 a) If E is measurable prove that E+~ is also measurable. Also prove that m$(E+\sim)$ = m(E).

b) Prove that there exists a non-measurable subset of R.

4 a) State and prove Factou's lemma.

b) State and prove monotone convergence theorem.

5 a) Suppose f, g are bounded measurable functions defined on a measurable set E of finite measure. Prove that

i) $\int_E (af + bg) = a\int_E f + b\int_E g$ for all constants a, b

ii) f = g a.e $\Rightarrow \int_E f = \int_E g$

b) State and prove bounded convergence theorem.

6 a) Suppose f is a real valued function defined on [a,b]. Prove that f is a function of bounded variation on [a,b] if and only if f can be expressed as a difference of the monotonically increasing functions on [a,b].

b) Suppose f is absolutely continuous on [a,b]. Prove that f is a function of bounded variation on [a,b].

7 Prove that L^P spaces are complete.

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8 a) Suppose (x,) is a measurable space and $E \subset X$. Prove that E is -measurable if and only if $\cdot_E$ is a –measurable function.

b) Suppose (x,) is a measurable space and f is a non-negative -measurable function defined on X. Prove that there exists a sequence $\{¨_n\}$ of simple functions such that
$0 \leq ¨_1 \leq ¨_2 \leq \ldots ¨_n \leq ¨_{n+1} \leq \ldots$ and $\lim_{n \to \infty} ¨_n = f$.

9 a) Define a signed measure ¬ on a measurable space. Suppose (x, ,«) is a measure space and f is an integrable function on X with respect to μ. Suppose ¬ (E) = $\int_E f$ du ∀ E ϵ . Prove that ¬ is a signed measure on .

b) State and prove Hahn-Decomposition theorem.

10 State and prove the Caraotheodory extension theorem.

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Code No. 2876/CDE

FACULTY OF SCIENCE

M. Sc. (Final) (CDE) Examination, July 2018

Subject : Statistics

Paper – II : Linear Models and Design of Experiments

Time : 3 Hours **Max. Marks: 80**

Note : Answer any five questions from Part–A, each question carries 4 marks, Answer all questions in Part–B, each question carries 12 marks.

PART – A (5 x 4 = 20 Marks)
(Short Answer Type)

1 What is meant by a Gauss Markov set up? Discuss its assumptions.
2 Describe the procedure for step-wise regression.
3 Explain Fisher's least significance difference test.
4 Explain analysis of covariance. What is the difference between ANOVA and ANCOVA?
5 Explain the analysis of 2^3 factorial experiment.
6 Give a comparative study of split plot design and factorial design.
7 Define a Simple lattice Design.
8 What is meant by a PBIBD?

PART – B (5 x 12 = 60 Marks)
(Essay Answer Type)

9 (a) Explain Aitkin's generalized least square estimation. Prove that this estimator is the BLUE of β.

OR

(b) Explain the analysis of multiple regression models.

10 (a) Explain the analysis of LSD with one missing observation.

OR

(b) Explain the analysis of covariance for two-way classification.

11 (a) Construct 2 replicates of a 2^3 factorial experiment confounding. The effect BC in the first replicate and the effect AC in the second replicate. Write the ANOVA of this design

OR

(b) Explain the construction and analysis of one-half fraction of 2^4 factorial design.

12 (a) Discuss the intra block analysis of a BIBD.

OR

(b) Explain the analysis of Youden square design.

13 (a) Define Partially Balanced incomplete Design with two associate classes. State and prove its parametric relations.

OR

(b) Explain the analysis of RBD. Describe its advantages over CRD.

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FACULTY OF SCIENCE

M.Sc. Final (CDE) Examination, July 2018

Subject: Mathematics

Paper – III
Operation Research and Numerical Techniques

Time: 3 Hours **Max.Marks: 80**

PART – A (5x4 = 20 Marks)
[Short Answer Type]

Note: Answer any five of the following questions in not exceeding 20 lines each.

1 Solve the following LPD graphically

Max $Z = 8000\ x_1 + 7000\ x_2$

STC $3x_1 + x_2 \leq 66$

$x_1 + x_2 \leq 45$

$x_1 \leq 20$

$x_2 \leq 40$

2 State and prove reduction theorem of assignment problem.

3 Define pure and mixed strategies in games. Also define a saddle point.

4 Explain the following terms in PERT:
 i) Optimistic time
 ii) Normal time
 iii) Pessimistic time

5 Explain Bisection method for finding a real root of equation f(x) = 0.

6 Explain Gauss elimination method for solving system of equations.

7 Derive Newton's forward difference interpolation formula.

8 Write the algorithm for Euler's method.

PART – B (5x12 = 60 Marks)
[Essay Answer Type]

Note: Answer the following questions in not exceeding four pages each, using internal choice.

9 a) Solve the following problem by simplex method.

Min $Z = x_1 = 3x_2 + 2x_3$

STC $3x_1 - x_2 + 3x_3 \leq 7$

$-2x_1 + 4x_2 \leq 12$

$-4x_1 + 3x_2 + 8x_3 \leq 10$

$x_1, x_2, x_3 \geq 10$

OR

b) Solve the following assignment problem

JOBS

	I	II	III	IV	IV
A	45	30	65	40	55
B	50	30	25	60	30
C	25	20	15	20	40
D	35	25	30	30	20
E	80	60	60	70	50

10 a) Solve the following (2x4) game graphically

		B I	II	III	IV
A	I	2	2	3	-1
	II	4	3	2	6

OR

b) A project schedule has the following characteristics.

Activity	Time	Activity	Time
(1-2)	2	(4-8)	8
(1-4)	2	(5-6)	4
(1-7)	1	(6-9)	3
(2-3)	4	(7-8)	3
(3-6)	1	(8-9)	5
(4-5)	5		

Construct the PERT network and find critical path and time duration of the project.

11 a) Use Newton-Raphson method to obtain a root, correct to three decimal places of the equation $4(x - \sin x) = 1$.

OR

b) Solve the equations:

$$2x + 3y + z = 9$$
$$x + 2y + 3z = 6$$
$$3x + y + 2z = 8$$

by the factorization method.

12 a) Use Lagrange's interpolation formula to obtain a polynomial for the following data.

x	1	2	3	4
f(x)	2	4	8	128

OR

b) Evaluate $\int_{4}^{5.2} \log_e x$ using (i) Trapezoidal Rule (ii) Simpson's rule taking (h = 0.2).

13 a) Solve the following problem by dual simplex method

Max $Z = -3x_1 - 2x_2$

STC $x_1 + x_2 \geq 1$

$x_1 + x_2 \leq 7$

$x_1 + 2x_2 \leq 10$

$x_2 \leq 3$

$x_1, x_2 \geq 1$

OR

b) Solve $\frac{dy}{dx} = x+y$ numerically using Taylor's series method start from x=1, y=0 and carry to x=1.2 with h=0.1.

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Code No. 2867 / CDE / O / BL

FACULTY OF SCIENCE

M.Sc. Final (CDE) Examination, July 2018

Subject: Mathematics

Paper – III
Analytical Number Theory

Time: 3 Hours **Max.Marks: 100**

Note: Answer any five questions. All questions carry equal marks.

1 a) If x≥1 and ~ > 0, ~ ≠ 1, prove that $\sum_{n\le x} \pm_{\sim}(n) = \frac{s(\sim+1)}{\sim+1} x^{\sim+1} + 0(x\ \ )$ where = Max{1,α}.

b) For x > 1 prove that $\sum_{n\le x} ¨(n) = \frac{3}{®^2} x^2$ + 0 (x log x) and hence prove that average order of ¨ (n) is $\frac{3n}{®^2}$.

2 a) For all x ≥1 prove that $\sum_{n\le x} d(n)$ = x log x + (2c-1)(x) + $0\left(\sqrt{x}\right)$.

b) For x ≥ 2 prove that $\sum_{p\le x}\left[\frac{x}{p}\right]$ log p = x log x + 0(x), where the sum is extended over all primes p ≤ x.

3 Show that the following relations are logically equivalent:

a) $\lim_{x\to\infty} \frac{®(x)\ \log\ x}{x} = 1$

b) $\lim_{x\to\infty} \frac{^{-}(x)}{x} = 1$

c) $\lim_{x\to\infty} \frac{\Psi(x)}{x}$

4 a) Let F be a real or complex valued function defined on (0, ∞) and let G(x) = log x $\sum_{n\le x} F\left(\frac{x}{n}\right)$. Then prove that F(x) log x $+\sum_{n\le x} F\left(\frac{x}{n}\right) \wedge (n) = \sum_{d/x} «(d)\, G\left(\frac{x}{d}\right)$.

b) Prove that for x > 0 we have · (x) log x + $\sum_{n\le x} \wedge(n) \cdot \left(\frac{x}{n}\right)$ = 2x log x + 0(x).

5 a) Show that a finite abelian group of order n has exactly n distinct characters.

b) State and prove orthogonality relations for characters.

6 For any real-valued non-principal character ¡ mod K let A(n) = $\sum_{d/n} ¡(d)$ and

B(x) = $\sum_{n\le x} \frac{A(n)}{\sqrt{n}}$. Then prove that

a) B(x) $\to \infty$ as x$\to \infty$

b) B(x) = 2$\sqrt{x}$ L(1, ¡) + 0(1) for all x ≥ 1.

Hence prove that L(1, ¡)≠0.

7 a) For x > 1 prove that

$$\sum_{p\le x}\frac{\log P}{P}=\frac{1}{¨(k)}\log x+\frac{1}{¨(k)}\sum_{r=2}^{¨(k)}\bar{·}_r(h)\sum_{p\le x}\frac{·_r(P)\log P}{\text{P}}+0(1)$$

P≡h (Mod K)

b) For x > 1, · ≠ · $_1$ Prove that

$$\sum_{P\le x}\frac{·(P)\log P}{\text{P}}=-L'(1,·)\sum_{\text{n}\le \text{x}}\frac{«(\text{n})·(\text{n})}{\text{n}}+0(1)$$

8 Let f be a multiplicative arithmetical function such that the series Σf(n) is absolutely convergent. Then show that the sum of the series can be expressed as an absolutely convergent infinite product

$$\sum_{n=1}^{\infty}f(n)=\underset{P}{®}\left\{1+f(p)+f(p^2)+...\right\}$$ extended over all primes.

Hence prove that

$$\frac{¶(s-1)}{¶(s)}=\sum_{n=1}^{\infty}\frac{¨(n)}{n^s}=\underset{P}{®}\left(\frac{1-P^{-s}}{1-p^{1-s}}\right)$$ if ± > 2.

9 For ± > 1 prove that

$$\Gamma(s)¸(s,a)=\int_o^{\infty}\frac{x^{s-1}e^{-ax}}{1-e^{-x}}\text{dx}$$

In particular for a = 1, prove that

$$\Gamma(s)¸(s)=\int_o^{\infty}\frac{x^{s-1}.e^{-x}}{1-e^{-x}}\text{dx.}$$

10 If c > 1 and x ≥ 1 prove that

$$\frac{·_1(x)}{x^2}=\frac{1}{2®\,i}\int_{c-i\infty}^{c+i\infty}\frac{x^{s-1}}{s(s+1)}\left(-\frac{¸'(s)}{¸(s)}\right)ds$$

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Code No. 2877/CDE

FACULTY OF SCIENCE

M. Sc. (Final) (CDE) Examination, July 2018

Subject : Statistics

Paper – III : Operation Research

Time : 3 Hours **Max. Marks: 80**

Note : Answer any five questions from Part–A, each question carries 4 marks, Answer all questions in Part–B, each question carries 12 marks.

PART – A (5 x 4 = 20 Marks)
(Short Answer Type)

1 What is the role of operations research in decision making?

2 Explain dual simplex method.

3 The activities along with their dependence relationships are given below. Draw an arrow diagram.

Activity	A	B	C	D	E	F	G	H	I
Immediate Predecessor	-	-	-	C	A, B	C	E, F	D	F, H

4 Explain the terms:
(i) Two-person-zero sum game
(ii) Pure strategy
(iii) Mixed strategies
(iv) Value of the game

5 Explain briefly various costs associated with inventory systems with suitable examples.

6 Explain different operating characteristics of a queuing system.

7 What is Integer linear programming? How does the optimal solution of an I.P.P. can be compared with that of L.P.P?

8 What is a sequencing problem? How a three machine problem is converted to 2 machine problem? Explain.

PART – B (5 x 12 = 60 Marks)
(Essay Answer Type)

9 (a) Explain Dual Simplex algorithm.

OR

(b) Explain the basic difference between sensitivity analysis and parametric programming. Also discuss the effect of discrete changes in the cost vector C under sensitivity analysis.

10 (a) Define a T.P.P. Solve the Transportation problem for optimal cost

	D_1	D_2	D_3	D_4	Supply
O_1	1	2	2	3	70
O_2	2	4	0	1	38
O_3	1	2	2	5	32
Demand	40	28	30	42	

OR

(b) What is dominance? Apply it and solve the following game.

$$A \overset{B}{\begin{bmatrix} 4 & -1 & 4 & 3 \\ 1 & 15 & 7 & 8 \\ 1 & 8 & 7 & 6 \\ 0 & 9 & -10 & 5 \end{bmatrix}}$$

11 (a) Define queuing problem. If the number of arrivals in some time interval follows Poisson distribution, show that the distribution of the time interval between two consecutive arrivals is exponential.

OR

(b) Derive s – S policy with exponential demand.

12 (a) What is dynamic programming problem? Explain how a L.P.P. is solved by Dynamic programming Technique.

OR

(b) Explain about different models considered under goal programming problem.

13 (a) Derive an Inventory model for perishable items.

OR

(b) Explain the Gomory's cutting plane all integers algorithm of an I.P.P.

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Code No. 2863 / CDE

FACULTY OF SCIENCE

M.Sc. Final (CDE) Examination, June / July 2018

Subject: Mathematics

Paper – IV
Fluid Mechanics

Time: 3 Hours **Max.Marks: 80**

PART – A (5x4 = 20 Marks)
[Short Answer Type]

Note: Answer any five of the following questions. Each question carries four marks.

1 State and prove conservation of linear momentum.

2 Find centre of mass of a thin wire bent into the form of semi-circular wire.

3 Discuss motion of liquid streaming past a fixed circular cylinder.

4 Discuss plane couette flow.

5 Write the elementary properties of vortex motion.

6 Define centre of vortices.

7 State the Buckingham ® –theorem.

8 What is boundary layer separation?

PART – B (5x12 = 60 Marks)
[Essay Answer Type]

Note: Answer all the following questions by using internal choice.
Each question carries 12 marks.

9 a) Show that $\frac{x^2}{a^2}\tan^2 t + \frac{y^2}{b^2}\cot^2 t = 1$ is a possible form of the boundary surface of a liquid and find an expression for the normal velocity.

OR

b) Sources of strength m are placed at (a,0) (-a,0) and a sink of strength 2m at (0,0), find stream formation and magnitude of velocity of the flow.

10 a) Discuss the motion of a cylinder in a uniform stream and find its stream lines.

OR

b) Derive Navier – Stoke's equation.

11 a) Discuss the motion of sphere through an infinite mass of a liquid at rest at infinity.

OR

b) Determine the stream function when the strength of the vortex filament are equal.

12 a) Obtain the boundary layer equation in two dimensional flow.

OR

b) Obtain the Karman's integral equation.

13 a) State and prove Blasius' theorem.

OR

b) Discuss steady flow between two co-axial cylinders.

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Code No. 2868 / CDE / O / BL

FACULTY OF SCIENCE

M.Sc. Final (CDE) Examination, June / July 2018

Subject: Mathematics

Paper – IV
Integral Transforms, Integral Equations & Calculas of Variations

Time: 3 Hours **Max.Marks: 100**

Note: Answer any five questions choosing atleast two from each part.

All questions carry equal marks.

PART – A

1 a) If L{F(t) = f(p), then show that L $\left[\frac{1}{t}F(t)\right] = \int_p^\infty f(x)\,dx$.

b) Find L$^{-1}\left[\frac{p-1}{(p+3)(p^2+2p+2)}\right]$.

c) Solve (D^4 + 2D^2 + 1) y=0, where y(0) = 0, y'(0) = 1, y''(0) = 0 and y'''(0) = 3.

2 a) Find the Fourier sin a transform of f(x) sin ax.

b) State and prove convolution theorem for Fourier transforms.

3 a) Form an integral equation corresponding to $\frac{d^2y}{dx^2}$ + y = cos x, y(0) = 0, y'(0)=1.

b) Find the resolvent kernel for volterra integral equation with Kernel k(x,t) = -2 + 3(x-t).

4 a) Solve the volterra integral equation using the method of successive approximation ¨ (x) = 2x^2 + 2 - $\int_o^x x$¨ (t) dt, ¨$_o(x)$ =2.

b) Solve the integral equation ¨ (x) = cos x + a$\int_o^{®}$ sin (x - t) ¨ (t) dt.

5 a) Solve the integro-differential equation
¨ $''$ (x) + 2 ¨ $'(x) - 2\int_o^x$ sin (x-t) ¨ $'$ (t) dt = cos x. ¨ (0) = ¨ $'$(0) = 0.

b) Solve the integral equation ¨ (x) = cos x + a$\int_o^{®}$ sin $(x-t)$ ¨ (t) dt.

CONT2…

PART – B

6 a) Solve the integral equation $\varphi(x) = 2\int_o^1 xt\,\varphi^3(t)\,dt$.

b) Solve the boundary value problem using Green's function $y'' + y = x$; $y(0) = y(\pi/2) = 0$.

7 a) Derive the necessary condition for the functional $v[y(x)] = \int_a^b F(x, y, y')\,dx$ with the boundary conditions $y(a) = y_a$, $y(b) = y_s$ to have an extremum.

b) Prove that the shortest distance between two points in a plane is a straight line.

8 a) Define Brachistochrone problem and obtain the solution of the same in the form of a cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$.

b) Find a curve with specified boundary points whose rotation about the axis of abscissa generates a surface of minimum area.

9 a) State isometric problem, and find the extremals of the problem

$v[y(x)] = \int_o^1 (y'^2 + x^2)\,dx$ given that $\int_o^1 y^2\,dx$; $y(0) = 0$, $y(1) = 0$.

b) Show that the sphere is the solid figure of revolution, which for a given surface area, has maximum volume.

10 a) Derive Lagrange equation of motion, using variational principle.

b) Use Lagrange equation of motion to find the equation of motion of the simple pendulum.

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Code No. 2878/CDE/N

FACULTY OF SCIENCE

M. Sc. (Final) (CDE) Examination, July 2018

Subject : Statistics

Paper – IV : Time Series Analysis Statistical Process & Quality Control

Time : 3 Hours **Max. Marks: 80**

Note : Answer any five questions from Part–A, each question carries 4 marks, Answer all questions in Part–B, each question carries 12 marks.

PART – A (5 x 4 = 20 Marks)
(Short Answer Type)

1 Explain briefly the autocovariance and autocorrelation of a stationary process.
2 Define ARMA (1, 1) model and explain its properties, stationarity and invertibility.
3 Define MA (Q) process.
4 Define ARIMA model. How AR, MA and ARMA models are obtained as particular case of it?
5 Distinguish between Assignables causes and chance causes of variation.
6 Explain CUSUM charts.
7 What are the advantages of applying accepting sampling procedures in an Industry?
8 Explain the designing of single sampling plan with given producer's risk and consumer's risk.

PART – B (5 x 12 = 60 Marks)
(Essay Answer Type)

9 (a) Describe the link between sample spectrum and ACF.

OR

(b) Describe the periodogram analysis in Time series.

10 (a) Explain the three explicit forms of ARIMA model.

OR

(b) Explain the method of updating forecasts for a given lead time.

11 (a) Explain the use of OC function and ARL function in control charts theory.

OR

(b) What are EWMA charts? Explain in what way they are different from ordinary moving average chart.

12 (a) Design a Double sampling plan given p_1, p_2, α and β.

OR

(b) What are Rectifying sampling plans? Derive Rectifying single sampling plan for specified AOQL and LTPD.

13 (a) What is a linear stationary model? Give the equivalent forms for the general linear process.

OR

(b) Explain the operation of V-mask, to study a two sided CUSUM scheme. Give the procedure to draw V-mask.

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FACULTY OF SCIENCE

M.Sc. Final (CDE) Examination, July 2018

Subject: Mathematics

Paper – V

Integral Transforms Integral Equations & Calculas of Variations

Time: 3 Hours **Max.Marks: 80**

PART – A (5x4 = 20 Marks)

[Short Answer Type]

Note: Answer any five of the following questions. Each question carries four marks.

1 Find $L[e^t \sin^2 t]$

2 Find $L^{-1}\left[\frac{1}{P^2+8P+16}\right]$

3 Define volterra and Fredholm integral equations.

4 Form integral equation corresponding to the differential equation $y''+(1+x^2)y = \cos x$, $y(0)=0$, $y'(0)=2$.

5 Define Green's function and write its four properties

6 Show that if Kernel k(x,t) is symmetric then all its iterated kernels are symmetric

7 Find the shortest distance between the points (x_1,y_1) and (x_2, y_2)

8 State the isoperimetric problem and indicate its solution.

PART – B (5x12 = 60 Marks)

[Essay Answer Type]

Note: Answer all the following questions by using internal choice.
Each question carries 12 marks.

9 a) State and prove convolution theorem of Laplace transform. Using it find $L^{-1}\left[\frac{P}{(P^2+a^2)^2}\right]$.

OR

b) i) Find the finite cosine transform of $\left(1-\frac{x}{®}\right)^2$.

ii) Find Fourier sine transform of $\frac{e^{-ax}}{x}$.

10 a) Solve the integro differential equation

$$¨''(x) + 2¤(x) - 2\int_o^x \sin(x-t)\,¨'(t)\,dt=\cos x,\ ¨(0)=¤(0)=0.$$

OR

b) Define Abel's problem and solve its integral equation.

11 a) Find the characteristic numbers and eigen functions of the Fredholm integral equation ¨ (x) = ª $\int_{o}^{®}$($\cos^2$ x cos 2t + cos 3x $\cos^3$ t) ¨ (t)dt.

OR

b) Solve the boundary value problem by using Green's function
y'' + y = x, y(0) = y($® / 2$) = 0.

12 a) Find the extremal of the functional
V[x(t), y(t)] = $\int_{o}^{1}\left(y'^2 + x'^2 + 2y\right)$dx subject to the conditions
x(0)=0, x(1)=1, y(0)=1, y(1) = 3/2.

OR

b) Derive Lagrange equation of motion, using it find the governing equation of simple pendulum.

13 a) Explain the method of successive approximation. Using it find the solution of
¨ (x) = $2x^2$ + 2 - $\int_{o}^{x} x$ ¨ (t) dt; ¨ $_{o}$ (x)=2.

OR

b) Solve the homogeneous integral equation
¨ (x) = ª $\int_{o}^{2®} \sin$ (x+t) ¨ (t) dt.

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Code No. 2879/CDE

FACULTY OF SCIENCE

M. Sc. (Final) (CDE) Examination, July 2018

Subject : Statistics (Practical)

Paper – I : Statistical Inference, Linear Models and Design of Experiments

Time : 3 Hours **Max. Marks: 100**

Note : Answer any three questions. All question carry equal marks.

1 Given below are data on expense ratio (in %) for 20 large-cap growth mutual funds: 0.52, 1.06, 1.26, 2.17, 1.55, 0.99, 1.1, 1.07, 1.81, 2.05, 0.91, 0.79, 1.39, 0.62, 1.52, 1.02, 1.1, 1.78, 1.01, 1.15. Assuming normal model (i) is there compelling evidence for concluding that the population mean expense ratio exceeds 1% ? Carryout a test of the relevant hypothesis using α=0.01. (ii) Draw the power curve with atleast five points (iii) What is α of the test that rejects H_o for $\overline{X}$ > 1.16?

2 (a) Construct SPRT taking α = 0.01, β = 0.1 for a N(1, 0.5) models and $H_o : \mu = 1.0$ against $H_1 : \mu = 1.15$ carryout the test for the data in Question No. 1 above and state the decision.

(b) Axial stiffness index for three different plate lengths are given below:

A (4") :	309.2	311.0	316.8	326.5	349.8
B (6") :	331.0	347.2	348.9	361.0	381.7
C (8") :	351.0	357.1	366.2	367.3	382.0

Carryout a non-parametric test for testing equality of mean stiffness index for the three lengths. Take α = 0.05.

3 The data below pertains to y : phosphate absorption index, x_1 : amount of extractable iron and x_2 : amount of extractable aluminum :

y	4	18	14	26	21	30	28	36	65
x_1	61	175	111	130	169	169	160	244	257
x_2	13	21	24	64	33	61	39	71	112

(i) Fit a least squares linear regression of y on x_1 and x_2 .
(ii) Compute R^2 and interpret the value.
(iii) Assuming normality, test the hypothesis

$$H_o : \begin{pmatrix} l \\ 2 \end{pmatrix} = \underline{0} \quad \text{vs.} \quad H_1 : \begin{pmatrix} l \\ 2 \end{pmatrix} \neq \underline{0}$$

(iv) Construct 95% confidence interval β_1.
(v) Predict the value of mean of y when x_1 = 150, x_2 = 50.

4 In an experiment to compare three different brands of pens and four different wash treatments with respect to their ability to remove marks on a particular type of fabric, indicator of overall specimen colour change was measured and are given below:

		Washing treatment			
		1	2	3	4
Brand of pen	A	0.97	0.48	0.48	0.46
	B	0.77	0.14	0.22	0.25
	C	0.67	0.39	0.57	0.19

(i) State the linear model to analyze the above data.
(ii) Assuming normality, test the relevant hypotheses.

5 The octane number is measured for 7 different gasolines (Latin letters). The design is a 3 x 7 Youden square. The Knak meter is read for 7 engines at 60, 90 and 120 seconds. The data are given below:

Time	1	2	3	4	5	6	7
60	A 43	B 36	C 33	D 44	E 41	F 36	G 33
90	B 34	C 32	D 47	E 40	F 35	G 32	A 41
120	D 47	E 46	F 43	G 33	A 44	B 32	C 27

Analyse the above data.

6 The data in the following table are from an experiment to study the effect of two supplements each at two levels (ℓ_o, ℓ_1, s_o, s_1) to a corn ration for feeding pigs of both gender (male: p_1, female : p_o). The experiment was carried out in 2 replicates with two incomplete blocks:

Replicate – I		Replicate – II	
Block – 1	Block-2	Block-3	Block-4
ℓ : 0.87	ℓp : 1.09	ℓps : 1.08	ℓp : 1.09
p : 1.48	(1) : 1.03	p : 1.22	ps : 1.45
ℓs : 1.22	ℓps : 1.38	ℓ : 1.00	ℓs : 1.13
ps : 1.52	s : 1.11	s : 0.97	(1) : 0.97

Identify the confounded effects and carryout the analysis of the data.

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Code No. 7960/CDE/N

FACULTY OF SCIENCE

M.Sc. (Final) CDE Examinations, July 2019

Subject: Mathematics

Paper- I : Complex Analysis

Time: 3 Hours **Max. Marks: 80**

PART – A (5 x 4 = 20 Marks)

(Short Answer Type)

Note: Answer any five of the following questions. Each question carries 4 marks.

1. Find the fixed points of the linear transformation $w = \frac{3z-4}{z-1}$
2. Find the linear transformation which carries 0, i, -i into 1, -1, 0 respectively.
3. Compute $\int_{¥} xdz,$ where γ is the directed line segment from 0 to 1+i.
4. If $f(z)$ and $g(z)$ are analytic in a region Ω, and if $f(z) = g(z)$ on a set which has an accumulation point in Ω, then prove that $f(z) = g(z)\ \forall\ z \in \Omega$
5. Find the poles and residues of the function $f(z) = \frac{1}{z^2 + 5z + 6}$
6. Find the number of roots of the equation $z^7 - 2z^5 + 6z^3 - z + 1 = 0$ in $|z| < 1$
7. Express $\sum_{n=-\infty}^{\infty} \frac{1}{z^2 - n^2}$ in the closed form
8. Show that $\Gamma\left(\frac{1}{6}\right) = 2^{-\frac{1}{3}}\left(\frac{3}{®}\right)^{\frac{1}{2}}\Gamma\left(\frac{1}{3}\right)^2$

PART – B (12 x 5 = 60 Marks)

(Essay Answer Type)

Note: Answer all the following questions by using internal choice. Each question carries 12 marks.

9. a) State and prove Lucas theorem.

OR

b) Find the radius of convergence of the following power series:

$\sum n!z^n, \sum q^{n^2} z^n (|q| < 1), \sum z^{n!}$

10. a) If the function $f(z)$ is on the rectangle $R = \{(x, y) : a \le x \le b, c \le y \le d\}$, then prove that $\int_{\partial R} f(z)dz = 0$

OR

b) State and prove the Taylor's theorem.

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11. a) If $f(z)$ is meromorphic in a region Ω with the zeros a_j, and the poles b_k, then prove that

$$\frac{1}{2\pi i}\int_{\gamma}\frac{f'(z)}{f(z)}dz=\sum_{j}n(\gamma,a_j)-\sum_{k}n(\gamma,b_k)$$

For every cycle γ which is homologous to zero in Ω and does not pass through any of the zeros on poles.

OR

b) Evaluate $\int_{o}^{\frac{\pi}{2}}\frac{dx}{a+\sin^2 x}\ \ (|a|>1)$

12. a) State and prove Laurent's theorem.

OR

b) Prove that $(2\pi)^{\frac{n-1}{2}}\Gamma(z)=n^{z-\frac{1}{2}}\Gamma\left(\frac{z}{n}\right)\Gamma\left(\frac{z+1}{n}\right)\ldots\ldots\ldots\ldots\Gamma\left(\frac{z+n-1}{n}\right).$

13. a) State and prove Jensen's formula.

OR

b) State and prove Schwarz's theorem.

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Code No. 7975/CDE

FACULTY OF SCIENCE

M.Sc. (Final) CDE Examination, July 2019

Subject : Statistics

Paper – I : Statistical Inference

Time : 3 Hours **Max. Marks: 80**

Note: Answer all questions from Part – A and Part – B. Each question Carries 4 marks in Part – A and 12 marks in Part – B.

PART – A (5x4 = 20 Marks)
(Short Answer Type)

1. Define family of distribution with monotone likelihood ratio (MLR). Give an example of a distribution having MLR property and an example of a distribution which does not satisfy MLR property.
2. A sample of size one is taken from Poisson (λ). To test H_o: λ = 2 against H_1: λ = 1 consider the test function $\ddot{}(x) = \begin{cases} 1 & if\ x \le 2 \\ 0 & otherwise \end{cases}$. Find the probability of type I, type II errors and power of the test.
3. State and prove Wald's fundamental identity.
4. Explain SPRT test procedure to test H_o:$\theta=\theta_o$ against the alternative H_1 : $\theta=\theta_1$ $(\theta>\theta_o)$for exponential distribution with mean θ.
5. Describe Wilcoxon Signed rank test.
6. Explain Chi-square test for goodness of fit.
7. Explain Hodges-Lehmann deficiency with an example.
8. Distinguish between Minimax and Bayes' decision functions. Describe admissible and optimal decision functions.

PART – B (5x12=60 Marks)
(Essay Answer Type)

9. (a) Define uniformly most powerful (UMP) test of a hypothesis. Based on a sample of size n from a distribution with pdf $f_\theta(x)=\theta x^{\theta-1}$, 0<x<1, θ>0, find a UMP size α test for testing H_o:$\theta \le \theta_o$ against H_o:$\theta>\theta_o$.

OR

(b) Describe likelihood ratio test (LRT). Establish relation between LRT and Neyman-Pearson test. Obtain asymptotic distribution of LRT.

10. (a) Obtain the bonds for constant of SPRT in terms of its strength (α¸β). Develop an SPRT of strength (α¸β) for testing H_o: $\sigma^2=\pm_0^2$ against H_1 : $\pm_2 = \pm_1^2$ where successive observations come from N(0, σ^2). Obtain OC function of the test.

OR

(b) Define OC and SN functions of SPRTY. Find the expressions for OC and ASN functions of SPRT with respect to the distribution P(X=-1)=θ, P(X=1)=1-θ, 0<θ<1.

11. (a) Describe two-sample location problem. Discuss in detail Mann-Whitney-Wilcoxon test for two-sample location problem.

OR

(b) Describe Spearman's and Kendalls tests of independence.

12. (a) Prove that, under squared error loss function, mean of the posterior distribution is a Bayes estimator. Suppose X is distributed as $B(1,\theta)$ and prior distribution of θ is $\pi(\theta)=1$, find the Bayes estimator of θ under squared error loss function.

OR

(b) Define risk function under squared error loss function. Find Bayes risk if X has $B(n,\theta)$ distribution and prior distribution of θ is Beta(α¸β).

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Code No. 7965/CDE/BL

FACULTY OF SCIENCE

M.Sc. (Final) CDE Examinations, July 2019

Subject: Mathematics

Paper- I : Topology & Functional Analysis

Time: 3 Hours **Max. Marks: 100**

PART – A

(Short Answer Type)

Note: Answer any five of the following questions, choosing atleast two from each Part. All questions carry equal marks.

1. Let X be a non-empty set and let there be given a 'closure' operator which assigns to each subset A of X a subset $\overline{A}$ of X in such a manner that $(a)\ \overline{¤} = ¤ \quad (b) A \subseteq \overline{A} \quad (c) \overline{\overline{A}} = \overline{A} \quad (d) \overline{A \cup B} = \overline{A} U \overline{B}$

   If a closed set A is defined to be one for which $A = \overline{A}$, then show that the class of all complements of closed sets is a topology on X whose closure operation is precisely that initially given.

2. (a) Show that a metric space is sequentially compact if and only if it has the Bolzano Weierstress property.
   (b) Show that every sequentially compact metric space is sequentially metric space is totally bounded.
3. State and prove Ascoli's theorem.
4. State and prove Urysohn's Lemma.
5. a Show that continuous image of a connected .
   b) Prove that $\mathbb{R}^n$ and $\mathfrak{C}^n$are connected space is connected.

PART – B

(Essay Answer Type)

6. Let M be a closed linear subspace of a normed linear space N. If the norm of a coset x+M in the quotient space $\frac{N}{M}$ is defined by space is $\|x+M\| = imf\{\|x+m\| : m \in M\}$ then show that $\frac{N}{M}$ is a normed linear space. Further, Banach space show that $\frac{N}{M}$ is a Banach space.
7. State and prove Hahn-Banach Theorem.
8. a) State and prove Schwarz inequality.
   b) Show that inner product in a Hilbert space is jointly continuous.
9. If M is a closed linear subspace of a Hilbert. Space H then show that $H = M \oplus M^{\perp}$.
10. If $\{e_i\}$ is an orthonormal set in a Hilbert space H then show that $x - \sum (x, e_i) e_j \perp e_i$ for each j.

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Code No. 7961/CDE/N

FACULTY OF SCIENCE
M.Sc. (Final) CDE Examinations, July 2019

Subject: Mathematics
Paper: II - Measure Theory

Time: 3 Hours **Max. Marks: 80**

PART – A (5 x 4 = 20 Marks)
(Short Answer Type)

Note: Answer any five of the following questions. Each question carries 4 marks.

1. Suppose $\{E_j\}$ is a sequence of measurable sets such that $E_{j+1} \subset E_j \forall j$ and $m(E1) < \infty$ then prove that $m\left(\bigcap_{n=1}^{\infty} E\right) = \lim_{n\to\infty} m(E_n)$
2. Suppose f, g are bounded measurable functions defined on a measurable set E of finite measure prove that $\int_E (af + bg) = a\int_E f + b\int_E g$ for all constants a, b
3. Define positive variation P, Negative variation N and total variation T of a real valued function f defined on [a, b] prove that P+N =T
4. Suppose f is absolutely continuous on [a, b] prove that f is a function of bounded variation on [a, b]
5. State and prove Monotone convergence theorem.
6. Define a signed measure on a measurable space (x, β). Suppose(x, β, μ) is a measure space and f is an integrable function on X with respect to μ. Prove that the function $\nu : \beta \to \mathbf{R}$ defined by $\nu(E) = \int_E f \, d« \ \forall \ E \in$ is a signed measure on β.
7. Suppose μ^* is an outer measure on P(X) and β is the class of all μ^*-measurable sets. Prove that $\bar{«}$ the restriction of μ^* to β is a complete measure onβ .
8. If $E \subset F$ prove that $«_*(E) \le «_*(F)$.

PART – B (5x12 = 60 Marks)
[Essay Answer Type]

9. a) State and prove Egoroff's theorem.

OR

b) Suppose f is a non-negative measurable function which is integrable over a measurable set E. Prove that given any $\in$>0 there exists a $¢ > 0$ such that for all subsets $A \subset E$ with m(A)< δ we have $\int_A f < \in$

10. a) suppose f is a bounded measurable function defined on [a, b]. Suppose F(x) is defined by $F(x) = F(a) + \int_a^x f(t)dt$ for all $x \in [a,b]$. Prove that $F' = f.a.e$

OR

b) i) Prove that L^p [0, 1] is a linear space.

…2

ii) State and Prove Minkowski's inequality.

11. a) State and prove Fatou' s Lemma in a measure space (X, ,«).

OR

b. State and prove Lebesgue decomposition theorem.

12. a) State and prove Fubini's theorem.

OR

b) Suppose $E \subset X$ with $\mu^*(E) < \infty$. Prove that E is μ^*-measurable if and only if $« *(E) = «_*(E)$

13. a) Define a lobesgue measurable function.

Suppose f is an extended red valued function defined on a measurable subset E of **R**, prove that the following are equivalent.

i) $\{x \in E : f(x) > \sim\}$ is measurable for any $\alpha \in$ **R**

ii) $\{x \in E : f(x) \geq \sim\}$ is measurable for any $\alpha \in$ **R**

iii) $\{x \in E : f(x) < \sim\}$ is measurable for any $\alpha \in$ **R**

iv) $\{x \in E : f(x) \leq \sim\}$ is measurable for any $\alpha \in$ **R.**

b) Prove that the class β of all μ^*- measurable sets is a σ algebra of subsets of X.

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Code No. 7966/CDE/BL

FACULTY OF SCIENCE

M.Sc. (Final) CDE Examinations, July 2019

Subject: Mathematics

Paper – II : Measure & Integration

Time: 3 Hours **Max. Marks: 100**

PART – A

(Short Answer Type)

Note: Answer any five of the following questions, choosing atleast two from each Part. All questions carry equal marks.

1. a) Suppose $\mathcal{A}$ is a collection of subsets of X satisfying

   i) $A \in \mathcal{A} \Rightarrow \overline{A} \in A$

   ii) A, B $\in \mathcal{A} \Rightarrow$ A $\cap$ B $\in \mathcal{A}$ $\Rightarrow$ A - B $\in \mathcal{A}$

   Prove that A is an algebra of sets in X. Also prove that A, B $\in \mathcal{A}$

   b) Suppose $\mathcal{A}$ is an algebra of sets in X. Suppose $\{A_n\}$ is a sequence of sets in $\mathcal{A}$. Prove that there exists a sequence $\{B_n\}$ of sets in $\mathcal{A}$ such that

   i) $\overset{\infty}{\underset{n=1}{U}} B_n == \overset{\infty}{\underset{n=1}{U}} A_n$

   ii) $B_n \cap B_m = \ddot{} \quad for \quad n \neq m$

2. a) Prove that every Borel set is measurable.

   b) Suppose $\{f_n\}$ is a sequence of measurable functions, defined on a measurable set E in $\mathbb{R}$. Prove that

   i) $\sup\limits_n t_n$ ii) $\inf f_n$ iii) $\lim\sup f_n$ iv) lim inf f are all measurable functions.

3. a) Define Lebesque outer measure m*. Prove that m*. is countably sub additive.

   b) Suppose E $\subseteq$ **R**. Prove that the following are equivalent

   i) E is measurable

   ii) Given any $\in > 0$ there exists a closed set F such that F$\subset$E and m*(E-F)=0

4. a) Suppose f is a bounded function defined on a measurable set E of finite measure. Prove that f is measurable if and only if

$$\sup_{\substack{¤\leq f \\ ¤ simple}} \int_E ¤\, dx = \inf_{\substack{\cdot \geq f \\ \cdot\, simple}} \int_E \cdot \;\, dx$$

5. a) Suppose ϕ, ψ are simple functions which vanishes outside a set of finite measure. Prove that.

$$\int \left(a¤ + b\,\cdot\,\right) = a\int ¤ + b\int \cdot \;\, dx.$$

b. Suppose $\{u_n\}$ is a sequence of non-negative measurable functions defined on a measurable set E such that $\int_E f = \sum_{n=1}^{\infty} \int_E u_n$

PART – B
(Essay Answer Type)

6. a) Suppose f is integrable on [a, b] and $F(x) = \int_a^x f(t)\,dt \;\forall x \in [a,b]$ prove that F is a continuous function of bounded variation on [a, b]

b) Suppose f is integrable on [a, b] and $F(x) = F(a) + \int_a^x f(t)\,dt \;\forall x \in [a,b]$ prove that $F' = f \;a.e.$

7. Prove that a normed linear space X is complete if and only if every absolutely summable series is summable.

8. a) Suppose(X, β, «) is a measure space and f, g are non-negative measurable functions defined on E∈ β. Prove that $\int_E (af + bg) = a\int_E f + b\int_E g$ ∀ non-negative constants a, b.

b) State and prove Lebesque-Convergence theorem.

9. State and prove the Radon-Nikodym theorem.

10. Define an outer measure μ* and a μ*-measurable set ; Prove that the class β of all μ*-measurable sets is a σ-algebra of sets.

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Code No.7976/CDE

FACULTY OF SCIENCE

M.Sc. (Final) CDE Examination, June/July 2019

Subject : Statistics

Paper – II : Linear Models and Design of Experiments

Time : 3 Hours **Max. Marks: 80**

Note: Answer any five questions from Part – A and all questions from Part – B.

PART – A (5x4 = 20 Marks)
(Short Answer Type)

1. State and prove the necessary and sufficient conditions for estimability of a linear parametric function.
2. Explain the need for generalization of Linear Models.
3. Explain the basic principles of Design of Experiments.
4. Explain the analysis of CRD.
5. Explain the analysis of 2^2 factorial experiments.
6. Distinguish between Total confounding and Partial confounding.
7. What is BIBD? Explain.
8. What is Youden square design? Explain.

PART – B (5x12=60 Marks)
(Essay Answer Type)

9. (a) Show that the number of linear unbiased estimates (l.u.e's) of a linear parametric function (l.p.f) is either only one or infinitely many.

OR

(b) State and prove Gauss-Markov Theorem.

10. (a) Explain the analysis of covariance for one way classification.

OR

(b) Explain the analysis of LSD.

11. (a) Explain the analysis of 3^2 factorial experiments.

OR

(b) What are split-plot designs? When do you recommend the use of such designs? Outline the method of analysis of a split-plot design for two factors A and B having α and β levels respectively. Give the expressions for standard errors of differences of treatment means.

12. (a) Define simple lattice design with an example. Outline the analysis of this design.

OR

(b) When an incomplete block design is said to be balanced? For a balanced incomplete block design with the usual notations, prove that bk=vr, r(k-1)=λ(v-1) and b$\geq$v.

13. (a) Explain the analysis of RBD with one missing observation.

OR

(b) Construct one-quarter replicate of 2^5 factorial experiment with defining relations I_1= +ABCD and I_2= +ACE. Give the alias structure and analysis of this design.

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Code No. 7963/CDE/N

FACULTY OF SCIENCE

M.Sc. (Final) CDE Examination, July/August 2019

Subject : Mathematics
Paper – IV : Fluid Mechanics

Time : 3 Hours **Max. Marks: 80**

Note: Answer all questions from Part – A and Part – B. Each question Carries 4 marks in Part – A and 12 marks in Part – B.

PART – A (5x4 = 20 Marks)
(Short Answer Type)
Note: Answer any five of the following questions.
Each question carries four marks.

1. Describe Eulerian method.
2. Define centre of mass. Find centre of mass of solid hemisphere of radius a.
3. Discuss general motion of a cylinder in two dimensions.
4. Define elliptic co-ordinates.
5. Write elementary properties of Vortex Motion.
6. Show that every Vortex is always composed of the same elements of the fluid.
7. Define Reynold's number and write its significances.
8. Write a short note on Prandtl's boundary layer theory.

PART – B (5x12=60 Marks)
(Essay Answer Type)
Note: Answer all the following questions by using internal choice.
Each question carries 12 marks.

9. (a) State and prove Parallel axis theorem, using it find the moment of inertia of a uniform circular disc about an axis in the plane of the disc and tangent to the edge.

OR

(b) Derive equation of continuity in cylindrical coordinates.

10. (a) Discuss steady flow between two co-axial cylinders.

OR

(b) State and prove Blasius theorem.

11. (a) Discuss steady flow of an incompressible viscous fluid through a tube of elliptic cross section under constant pressure gradient.

OR

(b) Discuss Motion of sphere through a liquid at rest.

12. (a) Discuss unsteady flow of Viscous incompressible fluid over a suddenly accelerated flat plate.

OR

(b) Write short notes on the following
   (i) Dynamical similarity.
   (ii) Boundary layer thickness.
   (iii) Energy thickness.

13. (a) Derive Navier-Stoke's equations.

OR

(b) Show that $\frac{x^2}{a^2}\tan^2 t + \frac{y^2}{b^2}\text{cat}^2 t=1$ is a possible form of the boundary surface of a liquid and find an expression for the normal velocity.

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Code No. 7968/CDE/BL

FACULTY OF SCIENCE

M.Sc. (Final) CDE Examination, July/August 2019

Subject : Mathematics

Paper – IV : Integral Transforms Integral Equations & Calculas of Variations

Time : 3 Hours **Max. Marks: 100**

Note: Answer any Five questions choosing atleast two from each part.
All questions carry equal marks.

PART – A

1. (a) Fiond the Laplace transforms of G(t), where $G(t) = \begin{cases} e^{t-a}, & t > a \\ 0, & t < a \end{cases}$.

   (b) Evaluate $L^{-1}\left\{\frac{3p-2}{p^2+4p+20}\right\}$.

   (c) Solve $[tD^2+(1-2t)D-2]y=0$ if $y(0)=1$, $y'(0)=2$.

2. (a) Find Fourier sine transform of $\frac{e^{-ax}}{x}$.

   (b) Find the Hankel transform of e^{-ax} taking x $J_0(px)$ as the Kernel of the transformation.

3. (a) Form integral equation corresponding to the differential equation
   $y''+(1+x^2)y = \text{Cos } x$, $y(0) = y'(0)=0$.

4. (a) Solve the integral equation $¨(x) = e^2 + 2\int_0^x \cos(x-t)¨(t)dt.$

   (b) Solve the integral equation $\int_0^x \cos(x-t)¨(t)dt = x + x^2$.

5. (a) Solve the integral equation $¨(x) = \int_0^1 \cos(xt)¨^3(t)dt.$

   (b) Solve $¨(x) = Cos\ x + a\int_0^{®} \sin(x-t)¨(t)dt$ with degenerate Kernel.

PART – B

6. (a) Solve the homogeneous integral equation
   $$¨(x) = a\int_0^{®} Cos(x+t)¨(t)dt = 0.$$

   (b) Define Green's function and give its four properties.

7. (a) Derive the necessary condition for the functional $¬[y(x)] = \int_a^b F(x,y,y')\ dx$ with boundary conditions $y(a)=y_a$, $y(b)=y_b$ to have an extremum.

(b) Solve the variational problem $\int_0^e (xy' + yy')dx,\ y(0) = 0,\ \ y(e) = 1.$

8. (a) Find the external of the functional V[x(t), y(t)]= $\int_0^1 (y'^2 + x'^2 + 2y)dx$, subject to the conditions x(0)=0, x(1)=1, y(0)=1, y(1)=3/2.
   (b) State the isoperimetric problem and obtain its solution using principle of variational calculus.

9. (a) Find the extremats of the functional V[y(x), z(x)]= $\int_0^{®/2} [y'^2 + z'^2 + 2yz)dx$, y(0)=0=z(0), $y\left(®/2\right) = 1 = -z(®/2)$.
   (b) Prove that the sphere is the solid figure of revolution, which for given surface area, has maximum volume.

10. (a) State and prove Hamilton's principle, using it derive Lagrange equation of Motion.
    (b) Derive the equations of Motion of a projectile in space using Hamilton's equations.

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Code No.7978/CDE

FACULTY OF SCIENCE

M.Sc. (Final) CDE Examination, July/August 2019

Subject : Statistics

Paper – IV : Time Series Analysis Statistical Process & Quality Control

Time : 3 Hours **Max. Marks: 80**

Note: Answer any Five of the following in not exceeding 20 lines each.

PART – A (5x4 = 20 Marks)
(Short Answer Type)

1. Define Time series and give some real life examples.
2. How does a time series model identified using Auto correlation function and Partial auto correlation function.
3. Explain briefly the diagnostic checking of Time series model.
4. How do you obtain initial estimates of parameters for AR(p) model.
5. Explain the need of implementing control charts during production.
6. Explain the construction of R-chart.
7. What is an ideal OC-curve. How do you mark the producer's risk and consumer's risk on an ideal occurrence?
8. Define double sampling plan, write the advantages of double sampling plan over single sampling plan.

PART – B (5x12=60 Marks)
(Essay Answer Type)

9. (a) Explain the duality between AR and MA process.

OR

(b) Define power spectrum. Explain the estimation of power spectrum.

10. (a) Derive the minimum Mean square error forecasts.

OR

(b) Derive the MLE's of AR(p) model.

11. (a) What is the difference between C-chart and u-chart. Obtain the O.C. function of C-chart.

OR

(b) What is CUSUM chart. How do you calculate ARL in CUSUM chart.

12. (a) Explain Dodge continuous sampling plans and its properties.

OR

(b) Define ATI, Derive the formula for ATI.

13. (a) Derive the control limits for np-chart, when 'n' is varying.

(b) Derive the power spectrum and variance for ARMA (1, 1) model.

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Code No. 7964/CDE/N

FACULTY OF SCIENCE

M.Sc. (Final) CDE Examinations, July 2019

Subject: Mathematics

Paper - V : Integral Transforms Integral Equations & Calculus of Variations

Time: 3 Hours **Max. Marks: 80**

PART – A (5 x 4 = 20 Marks)

(Short Answer Type)

Note: Answer any five the following questions in not exceeding 20 lines each.

1. Evaluate L $\left\{e^{-2t}(3\cos 6t - 5\sin 6t)\right\}$
2. Find the finite Fourier Sine and cosine transform of $f(x) = x \quad in\ 0 < x < ®$.
3. Verify that $¨(x) = \frac{1}{®\sqrt{x}}$ is a solution of $\int_0^x \frac{¨(t)}{\sqrt{x-t}}dt = 1$
4. Solve the integral equation $¨(x)\sin x + 2\int_o^x e^{x-t}¨(t)dt$ by means of resolvent Kernels.
5. Find the resolvent Kernel for $k(x,t) = e^{x+t} \quad a = 0, \ b = 1$
6. Solve the following integral equation with degenerate Kernel.

$$¨(x) - 4\int_0^{\frac{®}{2}} \sin^2 x¨(t)dt = 2x - ®$$

7. Show that the arc length of the curve.

$l[y(x)] = \int_{x_0}^{x_1} \sqrt{1 + y^{12}}dx$ has externals on the straight line $y = C_1 x + C_2$

8. Find an extremum for the functional $\gamma[y(x)] = \int_{x_o}^{x_1}[y'' + 12xy]dx$ subject to $\int_{x_o}^{x_1} 6xdx$

PART – B (5 x 12 = 60 Marks)

(Essay Answer Type)

9. a) State and prove Convolution theorem for inverse Laplace transforms.

OR

b) State and prove convolution theorem for Fourier transforms.

10. a) Define Beta function and show that

(i) $(m,n) = \ (n,m) \ for \ m,n > 0$

(ii) $(m,n+1) + \ (m+1,n) = \ (\mathrm{m,n})$

OR

b) Illustrate the method for constructing a green's function for Second-order differential equation of the form.

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$[p(x)y']' + q(x)y = 0$ with boundary conditions $y(a) = y(b) = 0$

$p(x) \neq 0 on [a,b]$ & $p(x) \in C^{(1)}[a,b]$

11. a) Solve the integral equation $¤(n) - ª \int_{-®}^{®} (x\cos t + t^2 \sin x + \cos x \sin t) ¤(t) dt = x$

OR

b) Solve the following Integro-Differential Equation $¤''(x) + \int_o^x e^{2(x-t)} ¤'(t) dt = e^{2x}, ¤(0) = 0$,

$¤'(0) = 1$

12. a) Solve $¤(x) - ª \int_{-1}^{1} (5xt^3 + 4x^2 t + 3xt) ¤(t) dt = 0$

OR

b) Show that $(m,n) = 2\int_0^{\frac{®}{2}} Sin^{2m-1}x Cos^{2n-1}x dx$ hence evaluate $\int_0^{\frac{®}{2}} \sin^4 x \cos^6 x \, dx$.

13. a) Explain Brachitochrome problem and find a variational solution to it.

OR

(b) Derive Euler-Ostrogradsky Equation for the

function $v[z(x,y)] = \iint_D F\left(x, y, z, \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}\right) dxdy.$

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Code No. 7979/CDE

FACULTY OF SCIENCE

M.Sc. (Final Practical) CDE Examination, July 2019

Subject : Statistics

Paper – I : Statistical Inference, Linear Models & Design of Experiments

Time : 3 Hours **Max. Marks: 100**

Note: Answer any three questions. All questions carry equal marks.
(Scientific calculators are allowed)

1. (a) Five measurements of the tor content of a certain kind of cigarette yielded 14.5, 14.2, 14.4, 14.3 an d14.6 mg per cigarette. Show that the difference between the mean of this sample and the average tar claimed by the manufacturer, μ=14.0 is significant at α=0.05. Assume normality.
   (b) From the following data examine by using a sequential test procedure whether the coin is unbiased against the alternative that the probability of a head is 0.6. Take α=0.01 and β=0.1. For 30 trials the outcomes are : H T T H H H T H T H H H T H H T T H H H H T H T T H H T T T.

2. (a) The following random samples are measurements of the heat-producing capacity (in millions of calories per ton) of specimens of coal from two mines:

Mine 1:	8260	8130	8350	8070	8340	
Mine 2:	7950	7890	7900	8140	7920	7840

   Assuming the two populations from which the samples are drawn to be $N(«_1, ±^2)$ and $N(«_2, ±^2)$ respectively, test $H_0 : «_1 = «_2$ Vs $H_1 : «_1 \neq «_2$ at $\sim = 0.01$ level.
   (b) The following sequence of yes 'Y' and No. 'N' was obtained by individuals according to whether they are graduates or not : N N N N Y Y N Y Y N Y N N N N . Test for randomness of the series at 0.1 level.

3. Using the following data, test whether there is any association between disability of workers with their work performance. Test at $\sim = 0.05$.

		Performance		
		Above average	Average	Below average
Disability	Blind	21	64	17
	Deaf	16	19	14
	No disability	29	93	28

4. The following are sample data provided by a moving company on the weights of six shipments, the distances they are moved, and the damage that was incurred:

Weight(in 1000 pounds) X_1:	4.0	3.0	1.6	1.2	3.4	4.8
Distance(1000 miles) X_2:	1.5	2.2	1.0	2.0	0.8	1.6
Damage(Dollars) Y:	160	112	69	90	123	186

   a) Fit least squares linear regression of Y on X_1 and X_2.
   b) Test the hypothesis of overall goodness of fit.
   c) Test the hypothesis of the coefficient of X_2 is zero.

5. Seven different hardwood concentrations (A, B, C, D, E, F, G) are being studied to

determine the affect on the strength of the paper produced. However the plant can produce only three runs. The data was observed for a Youden square design.

Days(Blocks)

		1	2	3	4	5	6	7
Columns	1	A 114	B 120	C 117	D 149	E 210	F 119	G 117
	2	B 126	C 137	D 129	E 150	F 143	G 123	A 134
	3	D 141	E 145	F 120	G 136	A 118	B 118	C 127

(i) Carryout analysis to test the hypothesis of equality of mean strength for the seven hardwood concentrations.
(ii) Estimate the standard error of the difference between any two treatment effects.

6. An experiment was carried out to study the effect of N: nitrate (n_0, n_1), P: Potash (p_0, p_1) and phosphate K(k_0, k_1) each at two levels on the yield (in tons) of potatoes using two replicates with incomplete blocks confounding, one replicate is one replicate each:

Replicate-I

Block 1		Block 2	
Npr	68	(1)	67
n	70	np	75
p	90	nk	59
k	80	Pk	56

Replicate-II

Block 1		Block 2	
pr	78	np	64
npr	65	nr	77
(1)	55	p	102
n	48	k	58

Identify the confounded effects and carryout the analysis accordingly.

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Code No. 1054 / CDE

FACULTY OF SCIENCE

M.Sc. (Final) CDE Examination, November 2020

Sub: Mathematics

Paper – I: Complex Analysis

Time: 2 Hours **Max.Marks: 80**

PART – A (4x5 = 20 Marks)

Answer any four questions.

1 Show that an analytic function cannot have a constant absolute value without reducing to a constant.

2 Define cross ratio. Prove that cross ratio is invariant under a linear transformation.

3 Compute $\int_{|z|=1} \frac{e^{2}}{z} dz$.

4 If γ is the piecewise differentiable closed curve that does not pass through the point, a, then, prove that $\int_{\gamma} \frac{dz}{z-a}$ is the integral multiple of $2\pi i$.

5 Prove that an analytic function comes arbitrarily close to any complex value in every neighbourhood of an essential singularity.

6 If z=a is a pole of order m for f(z), then prove that

$$\operatorname{Res}_{z=a} f(z) = \frac{1}{(m-1)!} \lim_{z \to a} \frac{d^{m-1}}{dz^{m-1}} \left[(z-a)^m f(z) \right].$$

7 Prove that $\frac{\pi^2}{\sin^2 \pi z} = \sum_{n=-\infty}^{\infty} \frac{1}{(z-n)^2}$.

8 A necessary and sufficient condition for the absolute convergence of the product $\prod_{n=1}^{\infty} (1+a_n)$ is the convergence of the series $\sum_{n=1}^{\infty} |a_n|$.

PART – B (4x15 = 60 Marks)
Answer any four questions.

9 State and prove the necessary condition for analytic function.

10 Prove that the cross ratio (z_1, z_2, z_3, z_4) is real if and only if the four points lie on a circle or on a straight line.

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11 State and prove Cauchy's theorem for a rectangle.

12 i) State and prove Cauchy's integral formula.

ii) Evaluate $\int_{|z|=1} \frac{e^z}{z} dz$.

13 Evaluate $\int_0^{\pi/2} \frac{1}{1+\sin^2\theta} d\theta$

14 Evaluate $\int_0^{\infty} \frac{1}{1+x^4} dx$

15 State and prove argument principle.

16 State and prove the mean value property of harmonic functions.

17 State and prove Mittog-Leffler theorem.

18 State and prove Legendre's duplication formula.

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Code No.1059 / CDE / BL

FACULTY OF SCIENCE

M.Sc. (Final) CDE Examination, November 2020

Sub: Mathematics

Paper – I: Topology & Functional Analysis

Time: 2 Hours **Max.Marks: 100**

Answer any four questions. **(4x25 = 100 Marks)**

1 a) Prove that every second countable space is separable.
b) Prove that every separable metric space is second countable.

2 a) Prove that any closed subspace of a compact space is compact.
b) State and prove Lebesgue covering lemma.

3 State and prove Ascoli's theorem

4 a) Let X be a T_1 – space. Then prove that X is normal if and only if each neighbourhood of a closed set F contains the closure of some neighbourhood of F.
b) State and prove Urysohn's lemma.

5 a) Prove that a topological space X is disconnected if and only if there exists a continuous mapping of X onto the discrete two point space {0,1}.
b) Define component of a space. If $\{A_i\}$ is a non-empty class of connected subspaces of a topological space X such that $\cap_i A_i \neq \phi$, then prove that $A = \cup_i A_i$ is connected.

6 a) Let N be a normed linear space and x_0 be a non-zero vector of N. Then prove that there exists a bounded linear functional f_0 on N such that
$f_0(x_0) = \|x_0\|$ and $\|f_0\| = 1$.
b) State and prove closed graph theorem.

7 a) State and prove uniform boundedness theorem.
b) Prove that a non-empty subset X of a normed linear space N is bounded if and only if $f(x)$ is a bounded set of number for each $f \in N^*$.

8 a) Prove that a closed convex subset C of a Hilbert space H contains a unique vector of smallest norm.
b) Prove that if M is a proper closed linear subspace of a Hilbert space H, then there exists a non-zero vector z_0 in H such that $z_0 \perp M$.

9 Let H be a Hilbert space and let $\{e_i\}$ be an orthonormal set in H. Then prove that the following conditions are all equivalent to one another:

i) $\{e_i\}$ is complete

ii) $x \perp \{e_i\} \Rightarrow x = 0$

iii) If x is an arbitrary vector in H, then $x = \sum (x, e_i) e_i$.

iv) If x is an arbitrary vector in H, then $\|x\|^2 = \sum |(x, e_i)|^2$.

10 a) Prove that if T is an operator on Hilbert space H for which $(Tx, x) = 0$ for all x, then T=0.

b) If $p_1, p_2, \ldots, p_n$ are the projections on closed linear subspaces $M_1, M_2, \ldots, M_n$ of Hilbert space H, then prove that $P = P_1 + P_2 + \ldots P_n$ is a projection $\Leftrightarrow$ the $P_i's$ are pairwise orthogonal and in this case P is he projection on $M = M_1 + M_2 + \ldots M_n$.

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Code No. 1069 / CDE

FACULTY OF SCIENCE

M.Sc. (Final) CDE Examination, November 2020

Sub: Statistics

Paper – I: Statistical Inference

Time: 2 Hours **Max.Marks: 80**

PART – A (4x5 = 20 Marks)
Answer any four questions.

1 Define critical region and type-I, type-II errors.
2 Explain randomized and non-randomized tests.
3 What are the advantages of sequential analysis?
4 Define general form of OC function and give its importance.
5 Explain the concept of robustness in inference.
6 Explain one sample run test.
7 Explain ARE of one sample location test.
8 Define loss function and risk function and give their importance.

PART – B (4x15 = 60 Marks)
Answer any four questions.

9 Discuss consistency and unbiasedness of tests.

10 Distinguish between interval estimation and testing of hypothesis.

11 Show that SPRT terminates with probability one.

12 Explain SPRT procedure for binomial distribution and obtain OC, ASN functions.

13 Discuss Wilcoxon – Mann Whitney test for two sample problem.

14 Explain Chi-square test for goodness of fit and independence of attributes.

15 Explain estimation as a statistical decision problem.

16 Discuss about admissible decision function and optimal decision function.

17 Explain Ansari-Bradley two sample test procedure.

18 Derive OC and ASN functions in SPRT.

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Code No. 1055/CDE

FACULTY OF SCIENCE

M.Sc. (Final) CDE Examination, November 2020

Sub: Mathematics

Paper – II: Measure Theory

Time: 2 Hours **Max.Marks: 80**

PART – A (4x5 = 20 Marks)
Answer any four questions.

1 Define a σ – algebra of sets in X and furnish an example.

2 Prove that every countable set has outer measure zero.

3 Define absolute continuity of a function f defined on [a,b].

4 If f is of bounded variation on [a,b], then prove that $T_a^b = P_a^b + N_a^b$.

5 Define a measure space and give an example.

6 Define a positive set with respect to a signed measure and give an example.

7 Define a measurable rectangle.

8 Define an outer measure induced by a measure on an algebra.

PART – B (4x15 = 60 Marks)
Answer any four questions.

9 State and prove monotone convergence theorem.

10 State and prove Fatou's lemma.

11 State and prove Holder's inequality.

12 Prove that a normed linear space X is complete $\Leftrightarrow$ every absolutely summable series is summable.

13 State and prove Hahn's decomposition theorem.

14 State and prove Jordan's decomposition theorem.

15 State and prove Tonelli's theorem.

16 State and prove Caratheodory extension theorem.

17 If f is integrable on [a,b], then prove that the function defined by

$F(x) = \int_a^x f(t)\,dt$ is a continuous function of bounded variation on [a,b].

18 Prove that the L^P-spaces are complete.

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Code No. 1055/O/CDE/BL

FACULTY OF SCIENCE

M. Sc. (Final) CDE Examination, November 2020

Subject : Mathematics

Paper – II : Measures Theory

Time : 2 Hours **Max. Marks: 100**

Note: Answer any four questions. **(4x25=100 Marks)**

1 (a) State and prove monotone convergence theorem.
(b) State and prove Fatou's lemma.

2 (a) Construct a set which is non-measurable in the sense of Lebesgue.
(b) State and prove bounded convergence theorem.

3 (a) State and prove Holder's inequality.
(b) Prove that a normed linear space X is complete $\Leftrightarrow$ every absolutely summable series is summable.

4 Show that L^p – spaces are complete.

5 (a) State and prove Hahn's decomposition theorem.
(b) State and prove Jordan's decomposition theorem.

6 State and prove generalized Fatou's lemma.

7 State and prove Caratheodory extension theorem.

8 State and prove Fubini's theorem.

9 State and prove Riesz-Fischer theorem.

10 (a) If f is of bounded variation on [a, b], than prove that $T_a^b = P_a^b + N_a^b$.
(b) Prove that the set of all μ^* - measurable sets in X is a σ- algebra.

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Code No. 1070 / CDE

FACULTY OF SCIENCE

M.Sc. (Final) CDE Examination, November 2020

Sub: Statistics

Paper – II: Linear Models and Designs of Experiments

Time: 2 Hours **Max.Marks: 80**

PART – A (4x5 = 20 Marks)
Answer any four questions.

1 What is multiple regression model? How do you estimate its parameters?

2 Define multiple and partial correlation coefficients. How do you compute in case X_1 is dependent and X_2, X_3 independent variables?

3 Define Latin square design and give its layout for 6 treatments.

4 State Cochran's theorem on quadratic forms. Discuss its use in analysis of variance.

5 Explain the concept of confounding in factorial experiment.

6 Define main effects and interactions in 3^2 factorial experiments.

7 Define B.I.B.D. Also define two associate P.B.I.B.D's.

8 Define simple lattice design.

PART – B (4x15 = 60 Marks)
Answer any four questions.

9 What do you meant by selecting the best regression equation. Explain the following methods of selecting but regression equations:

i) All possible regression method using R^2 criterion.

ii) Stepwise regression procedure.

10 Explain Aitkens generalised least squares method.

11 Explain in detail principles of experimental designs.

12 Discuss the analysis of covariance of two way classification.

..2

13 Give the method of construction of blocks in 3^3 partially confounding experiment for confounding NP^2K and NP^2K^2 parts of NPK interaction one in each replication.

14 Explain plain factorial experiments, also their advantages and drawback. Give the method of analysis of 2^4 factorial experiment by Yate's algorithm.

15 Explain B.I.B.D. model estimation, also give the intrablock analysis.

16 Prove the parametric relations in partially balanced block designs and derive the treatment sum of square (adjusted) in P.B.I.B.D with 2 associate classes.

17 Explain split plot design and give the method of analysis by deriving the sum of squares.

18 Explain analysis of complete RBD.

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Code No. 1056 / CDE

FACULTY OF SCIENCE

M.Sc. (Final) CDE Examination, November 2020

Sub: Mathematics

Paper – III: Operation Research and Numerical Techniques

Time: 2 Hours **Max.Marks:80**

PART – A (4x5 = 20 Marks)
Answer any four questions.

1 Write the characteristics of the standard four of LPP. Also define surplus variable in LPP.
2 State and prove reduction theorem of assignment problem
3 Define:
i) Saddle point
ii) Optimal strategies
iii) Value of the game.
4 Write the rules for drawing network diagram.
5 Explain secant method.
6 Explain the partition method.
7 Construct a difference table from the following values:

x	1	2	3	4	5	6	7	8	9
f(x)	1	8	27	64	125	216	343	512	729

8 Derive Simpson's 1/3 rule for integration.

PART – B (4x15 = 60 Marks)
Answer any four questions.

9 Solve the LPP by two-phase method:
Max $Z = 5x_1 + 8x_2$
STC $3x_1 + 2x_2 \geq 3$
$x_1 + 4x_2 \geq 4$
$x_1 + x_2 \leq 5$ and
$x_1, x_2 \geq 0$

10 Find the optimum solution to the following transportation problem.

To

		A	B	C	D	E	ai
From	I	3	4	6	8	8	20
	II	2	10	1	5	30	30
	III	7	11	20	40	15	15
	IV	2	1	9	14	18	13
b's	→	40	6	8	18	6	

11 Solve the game graphically whose payoff matrix for the player A is given below:

B

		I	II
A	I	2	4
	II	2	3
	III	3	2
	IV	-2	6

12 A project has the following characteristics.

Activity	1-2	2-3	2-4	3-5	4-5	4-6	5-7	6-7	7-8	7-9	8-10	9-10
to:	1	1	1	3	2	3	4	6	2	5	1	3
tp	5	3	5	5	4	7	6	8	6	8	3	7
tm	15	2	3	4	3	5	5	7	4	6	2	5

Construct a PERT network. Find critical path and variance.

13 Perform five iterations of the Muller's method to find the root of the equation.

$$f(x) = \cos x - xe^{x} = 0$$

14 Find the solution, to three decimals, of the system

83x + 11y – 4z = 95
7x + 52y + 13z = 104
3x + 8y + 29z = 71

Using Gauss-Seidel Method.

15 Derive Newton's Interpolation formulae. From the following table, find the number of students obtaining less than 45 marks.

Marks	30-40	40-50	50-60	60-70	70-80
No. of students	31	42	51	35	31

16 Evaluate $\int_{4}^{52} \log x \, dx$ using

i)Trapezoidal Rule
ii) Simpson's Rule.

17 Solve the following Assignment problem.

Jobs

Machines	I	II	III	IV	V
A	11	10	18	5	9
B	14	13	12	19	6
C	5	3	4	2	4
D	15	18	17	9	12
E	10	11	19	6	14

18 Find the numerical solution of the simultaneous equations:

$$\frac{dx}{dt} + 2x + 3y = 0$$

$$3x + \frac{dy}{dt} + 2y = 2e^{2t}$$

with conditions at t=0, x=1, y=2 at the point t=0.1 by Tayler's method.

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Code No. 1048/O/BL

FACULTY OF SCIENCE

M. Sc. (Final) CDE Examination, November 2020

Subject : Mathematics

Paper – III : Elementary Number Theory

Time : 2 Hours **Max. Marks: 100**

Note: Answer any four questions. **(4x25=100)**

1 (a) If P_n is n^{th} prime, prove that the infinite series $\sum_{n=1}^{\infty} \frac{1}{p_n}$ diverges.

(b) State and prove division algorithm.

2 Prove that set of all multiplicative functions is a group under Dirichlet product of functions.

3 (a) State and prove Lagrange's theorem for polynomial congruences modulo P where P is a prime.

(b) State and prove Wolstenholme's theorem.

4 (a) State and prove principle of cross classification.

(b) Assume $m_1, m_2, \ldots m_r$ are relatively prime in pairs. Let $b_1, b_2, \ldots b_r$ be arbitrary integers and let $a_1, a_2, \ldots, a_r$ satisfy $(a_k, m_k) = 1$ for k = 1, 2, …. r . Then prove that the linear system of congruences

$$\begin{aligned} a_1 x &\equiv b_1 \pmod{m_1} \\ a_2 x &\equiv b_2 \pmod{m_2} \\ &\vdots \\ a_r x &\equiv b_r \pmod{m_r} \end{aligned}$$

has exactly one solution modulo $m_1 m_2 \ldots m_r$.

5 (a) State and prove Gauss Lemma.

(b) State and prove Quadratic reciprocity law.

6 (a) Let P be an odd prime and let d be any positive divisor of (P – 1). Then prove that in every reduced residue system mod P there are exactly $\phi(d)$ numbers "a" such that $\exp_p(a) = d$. In particular, when $d = \phi(P) = P - 1$ there are exactly $\phi(P - 1)$ primitive roots mod P.

(b) Let x be an odd integer. If $\alpha \geq 3$ prove that $x^{\phi(2\alpha)/2} \equiv 1 \pmod{2^{\alpha}}$ and hence show that there are no primitive roots mod 2^{α}.

7 (a) State and prove Reciprocity law for Jacobi symbols.

(b) If $n \geq 1$, prove that $P(n) < e^{k\sqrt{n}}$ where $K = \prod (2/3)^{1/2}$.

8 State and prove Euler's pentagonal number theorem.

9 For | x | < 1, prove that

$$\prod_{m=1}^{\infty} \frac{1}{1-x^m} = \sum_{n=0}^{\infty} P(n)x^n \quad \text{Where } P(0) = 1$$

10 State and prove Jacobi's triple product identity.

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Code No. 1071/CDE

FACULTY OF SCIENCE

M. Sc. (CDE) Examination, November 2020

Subject : Statistics

Paper – III : Operation Research

Time : 2 Hours **Max. Marks: 80**

Note: Answer any four questions. **(4x5=20 Marks)**

1 Define standard linear programming problem. Give the algorithm for its graphical solution.
2 Explain about models and modeling in OR.
3 What is an unbalanced transportation problem? Explain how to resolve it.
4 Explain Dominance property.
5 Write about factors affecting inventory control.
6 Define a game and explain operating characteristics of a queuing system.
7 What is goal programming problem? Explain, how it is different from linear programming problem.
8 What is sequencing problem? Give two examples.

PART – B

Note: Answer any four questions. **(4x15=60 Marks)**

9 Explain the concept of duality. Write Dual-Simplex algorithm.

10 Explain the rule of steepest ascent and 'θ' rule.

11 Explain forward pass and backward pass calculation procedures. Also define critical path.

12 Define Assignment problem. Give Hungarian algorithm to solve it.

13 Derive distribution of arrivals in a queuing theory.

14 Define different costs involved in inventory models. Derive the expressions for EoQ in an inventory model of single period without setup cost, instantaneous demand and units are continuous.

15 Define Integer programming problem. Give Gomery's cutting plane algorithm to solve a pure integer programming problem.

16 What is Dynamic programming problem. Explain about characteristics of Dynamic programming

17 Distinguish PERT and CPM.

18 For any 2 x 2 two-person zero-sum game without any Saddle point having the pay-off matrix for player

$$A \begin{matrix} & B_1 & B_2 \\ A_1 & a_{11} & a_{12} \\ A_2 & a_{21} & a_{22} \end{matrix}$$

the optimum mixed strategies are $S_A = \begin{bmatrix} A_1 & A_2 \\ p_1 & p_2 \end{bmatrix}$ and $S_B = \begin{bmatrix} B_1 & B_2 \\ q_1 & q_2 \end{bmatrix}$ are determined by

$\frac{P_1}{P_2} = \frac{a_{22} - a_{21}}{a_{11} - a_{12}}$, $\frac{q_1}{q_2} = \frac{a_{22} - a_{12}}{a_{11} - a_{21}}$, where $P_1 + P_2 = q_1 + q_2$. Then show that value of the game to A is given by $v = \frac{a_{11}a_{22} - a_{21}a_{12}}{a_{11} + a_{22} - (a_{12} + a_{21})}$

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Code No. 1057 / CDE

FACULTY OF SCIENCE

M.Sc. (Final) CDE Examination, November 2020

Sub: Mathematics

Paper – IV: Fluid Mechanics

Time: 2 Hours **Max.Marks: 80**

PART – A (4x5 = 20 Marks)
Answer any four questions.

1 Write Newton's Laws of motion in detail.
2 State and prove conservation of angular momentum.
3 Discuss uniform flow post a stationary cylinder.
4 Define elliptic coordinates.
5 Write elementary properties of Vortex motion.
6 Derive the differential equation of steady viscous flow in tube of uniform cross-section.
7 What is boundary layer separation?
8 Describe the concept of prandtl boundary layer theory.

PART – B (4x15 = 60 Marks)
Answer any four questions.

9 Derive equation of continuity in cylindrical coordinates.

10 Derive Euler's equation of motion in the form

$$\frac{d\overline{q}}{dt} + (\overline{q}.\nabla)\overline{q} = -\frac{1}{p}\nabla\overline{p} .$$

11 Discuss steady flow between two co-axial cylinders.

12 Derive Navier-Stoke's equation of motion.

13 Discuss the motion due to circular and rectilinear vertices.

14 Discuss the motion of fluid in a uniform triangular cross-section under constant pressure gradient.

15 Explain: i) Dynamical similarity, ii) Boundary layer thickness, iii) Energy thickness.

16 Derive Van Karman's integral equation.

17 State and prove Blasius theorem.

18 Discuss the motion of flow between two parallel plates.

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Code No. 1062 /O /BL

FACULTY OF SCIENCE

M.Sc. (Final) CDE Examination, November 2020

Sub: Mathematics

Paper – IV: Integral Transforms Integral Equations & Calculas of Variations

Time: 2 Hours **Max.Marks: 100**

Note: Answer any four questions. **(4x25=100)**

1 a) Prove that $L\left\{\frac{\sin t}{t}\right\} = \tan^{-1} P$.

b) Find $L^{-1}\left[\log\left(1 - \frac{1}{p^2}\right)\right]$.

c) Solve $y'' + y' + 4t\,y = 0$ if y(0) = 3, $y'(0) = 0$.

2 a) Find Fourier sine transform of $f(x) = \sin ax$.

b) Find the Hankel transform of e^{-ax}/x, taking $xJ_0(px)$ as the Kernel of the transformation.

3 a) Find an integral equation corresponding to the differential equation $y'' + y = \cos x$, $y(0) = y'(0) = 0$.

b) Find the resolvent Kernel of the volterra integral equation with Kernel e^{x-t}.

4 a) Solve the integral equation $\varphi(x) = 2\int_0^1 xt\,\varphi^3(t)\,dt$.

b) Solved the integral differential equation $\varphi(x) = e^x + 2\int_0^x \cos(x - t)\,\varphi(t)\,dt$.

5 a) Solve the integral differential equation

$$\varphi''(x) + 2\phi(x) - 2\int_0^x \sin(x - t)\,\varphi'(t)\,dt = \cos x, \varphi(0) = \varphi'(0) = 0.$$

b) Solve the Volterra integral equation using the method of successive approximation.

6 Solve the boundary value problem using Green's function

$$y'' + y = x^2, y(0) = y(\pi/2) = 0.$$

7 Show that all iterated Kernels of a symmetric Kernels are symmetric.

8 Find the extremal of the functional $V(y(x)) = \int_0^1 (1 + y''^2)\,dx$ subject to conditions y $y(0) = 0$, $y'(0) = y(1) = y'(1) = 1$.

..2

9 Define Brahistochrone problem and obtain the solution of the same in the form of a cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$

10 Derive the equation of motion of simple pendulum using Lagrange's equation.

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Code No. 1072/CDE

FACULTY OF SCIENCE

M.Sc. (Final) CDE Examination, November 2020

Sub: Statistics

Paper – IV : Time Series Analysis, Statistics Process and Quality Control

Time: 2 Hours **Max.Marks: 80**

PART – A (4x5 = 20 Marks)
Answer any four questions.

1 Explain weighted moving average smoothing method.
2 Define two equivalent forms of linear stationary process.
3 Define ARIMA (p, d, q) and ARIMA (1, 1, 1) processes.
4 What are initial estimates? What is the use of it?
5 Explain about average run length curve. Give its formula.
6 State advantages of control charts for attributes over control charts for variables.
7 Describe Rectifying Sampling Procedures. Give an example.
8 Describe multiple sampling plans.

PART – B (4x15 = 60 Marks)
Answer any four questions.

9 Describe in detail about periodogram analysis of time series.

10 Define Moving Average Process of order ‘q’ and derive its properties.

11 Derive general expressions for ψ – weights and π – weights.

12 Derive parameter estimates of Auto Regressive model by the method of maximum likelihood estimation.

13 Give a detailed explanation on economic designing of X-chart.

14 Derive the control limits for moving average chart for mean and range.

15 Derive expression for ‘n’ in sampling plans for variables when upper specification limit is given.

16 Explain how to design a sequential sampling plan can be designed.

17 Explain Holt and Holt-winter exponential smoothing methods.

18 Define exponentially weighted moving average. Derive control limits based on the same statistic.

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Code No. 1058 / CDE

FACULTY OF SCIENCE

M.Sc. (Final) CDE Examination, November 2020

Sub: Mathematics

Paper – V: Integral Transforms Integral Equations & Calculus of Variations

Time: 2 Hours **Max.Marks: 80**

PART – A (4x5 = 20 Marks)

Answer any four questions.

1 Prove that $L\left[\frac{\sin t}{t}\right] = \tan^{-1}(p)$.

2 Find Fourier sin e transform of e^{-x}.

3 Find the resolvent Kernel for Volterra integral equation with Kernel $k(x,t) = \frac{2+\cos x}{2+\cos t}$.

4 Solve the integral equation by using the method of successive approximation. $\varphi(x) = x + 1 - \int_o^x \varphi(t)\,dt$, $\varphi_0(x) = 1$.

5 Solve the integral equation $\varphi(x) = 2\int_0^1 xt \quad \varphi^3(t)dt$.

6 Solve the boundary value problem using Green's function $y'' + y = 0,\ y(0) = y\left(\frac{\pi}{2}\right) = 0$.

7 State and prove the fundamental lemma of calculus of variation.

8 What is the problem of the Brachisto chrone?

PART – B (4x15 = 60 Marks)

Answer any four questions.

9 i) Find $L^{-1}\left[\frac{p-1}{(p+3)(\mathrm{p}^2+2p+2)}\right]$.

ii) Solve $[tD^2 + (1-2t)D - 2] > 0 \ if \ \ y(0) = 1, y'(0) = 2$.

10 i) Find Fourier cosine transform of f(x) = sin x

ii) Find Hankel transform of e^{-ax} taking $xJ_0(px)$ as the Kernel of the transformation.

11 Transform the problem into integral equation $\frac{d^2y}{dx^2} + \lambda y = f(x),\ y(0) = 1,\ y'(0) = 0$

12 Solve the integro-differential equation

$$\varphi''(x) + \int_0^x e^{2(x-t)} \varphi'(t)\, dt = e^{2x}, \varphi(0) = 0, \varphi'(0) = 1.$$

13 Find the characteristic numbers and eigen functions of the integral equation.

$$\varphi(x) = \lambda \int_0^\pi \left(\cos^2 x \cos 2t + \cos 3x \cos^3 t\right) \varphi(t)\, dt = 0 .$$

14 Construct the Green's function for the homogeneous B.V.P.

$$y^{iv}(x) = 0, y(0) = y'(0) = y(1) = y'(1) = 0.$$

15 Explain the isoperimetric problem and find a variational solution to it.

16 Derive the Euler-Poisson equation.

17 Derive Hamilton's canonical equation of motion.

18 Solve the equation $\varphi(x) - \lambda \int_0^\pi \left(\cos^2 x \cos 2t\right) + \cos^3 t \cos 3x)\, \varphi(t)\, dt$ =0.

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Code No. 1073 / CDE

FACULTY OF SCIENCE

M.Sc. (Final Practical) CDE Examination, November 2020

Sub: Statistics

Practical Paper – I: Statistical Inference Linear Models & Design of Experiments

Time: 2 Hours **Max.Marks: 100**

Note: Answer any two questions. All questions carry equal marks. Scientific calculation and allowed.

1 a) An experimental engine operated for 24, 28, 21, 23, 32 and 22 minutes with a gallon of a certain kind of fuel. Assuming normality, test the hypothesis $H_0: \mu = 29$ Vs $H_1: \mu < 29$, where μ is the mean operative time. Take $\alpha = 0.01$.

b) On the assumption that the distribution is normal with variance unity, examine by a sequential ratio test procedure whether the mean is zero against the alternative that it is 0.5 for the data: 2.12, 1.32, 0.81, -1.91, -1.23, -0.17, -0.38, -0.03, -1.65, -0.35, -0.99, -0.67, -0.28, 0.40, -0.12, -0.10, -0.83, -0.37, -0.48. Take $\alpha = 0.05$ and $\beta = 0.15$.

2 a) The following are the average weekly losses of worker hours due to accidents in 10 industrial plants before and after a certain safety program was put into operation:

Before	45	73	46	124	33	57	83	34	26	17
After	36	60	44	119	35	51	77	29	24	11

Use the 0.05 level of significance to test whether the safety program is effective.

b) In an industrial production line, items are inspected periodically for defectives. The following is a sequence of defective items, D, and non-defective items, N produced by this production line.

DDNNN DNN DDNNNNN DDDNN DNNNN DND.

Carry out such test at $\alpha = 0.05$, to determine whether the defectives are occurring at random.

3 Carry out Friedman test for the following data on mileage of cars for type of gasoline and brand of cars.

		Car Brand		
		A	B	C
Tyre of gasoline	I	22.4	17.0	19.2
	II	20.8	19.4	20.2
	III	21.5	18.7	21.2

Test at α = 0.05.

4 The following data pertains to nitrous oxide (y), humidity (x_1) and temperature (x_2) of air in a location

y	0.9	0.91	0.96	0.89	1.15	0.77
x_1	72.4	41.6	34.3	35.1	8.3	72.2
x_2	76.3	70.3	77.1	68.0	66.8	77.7

i) Fit the model y = $\beta_o + \beta_2 x_2$ + ε using least squares method of estimation.

ii) Test a) H_o: $(\beta_1 \ \ \beta_2)' = (0 \ \ 0)'$ and b) H_o: β_2 = 0.

5 The following data are obtained on time of reaction for a chemical process. The experiment was conducted using a BIBD.

		Batch of raw materials (blocks)			
		B1	B2	B3	B4
Catalyst (Treatment)	1	73	74	-	71
	2	-	75	67	72
	3	73	75	68	
	4	75	-	72	75

Carryout the analysis and draw your conclusions. Find the standard error of the BLUE of any two treatment.

6 The following is the data on hours of operation of stepping switch. The experiment was conducted as a 2^3 factorial experiment confounding one effect in each replicate.

Replicate-I				Replicate-II			
Block 1		Block 2		Block 3		Block 4	
(1)	50	a	75	abc	68	(1)	48
ab	62	B	63	a	78	ab	56
c	71	ac	523	b	55	ac	45
abc	55	bc	48	c	62	bc	35

Identify the confounded effects and carry out the analysis.

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